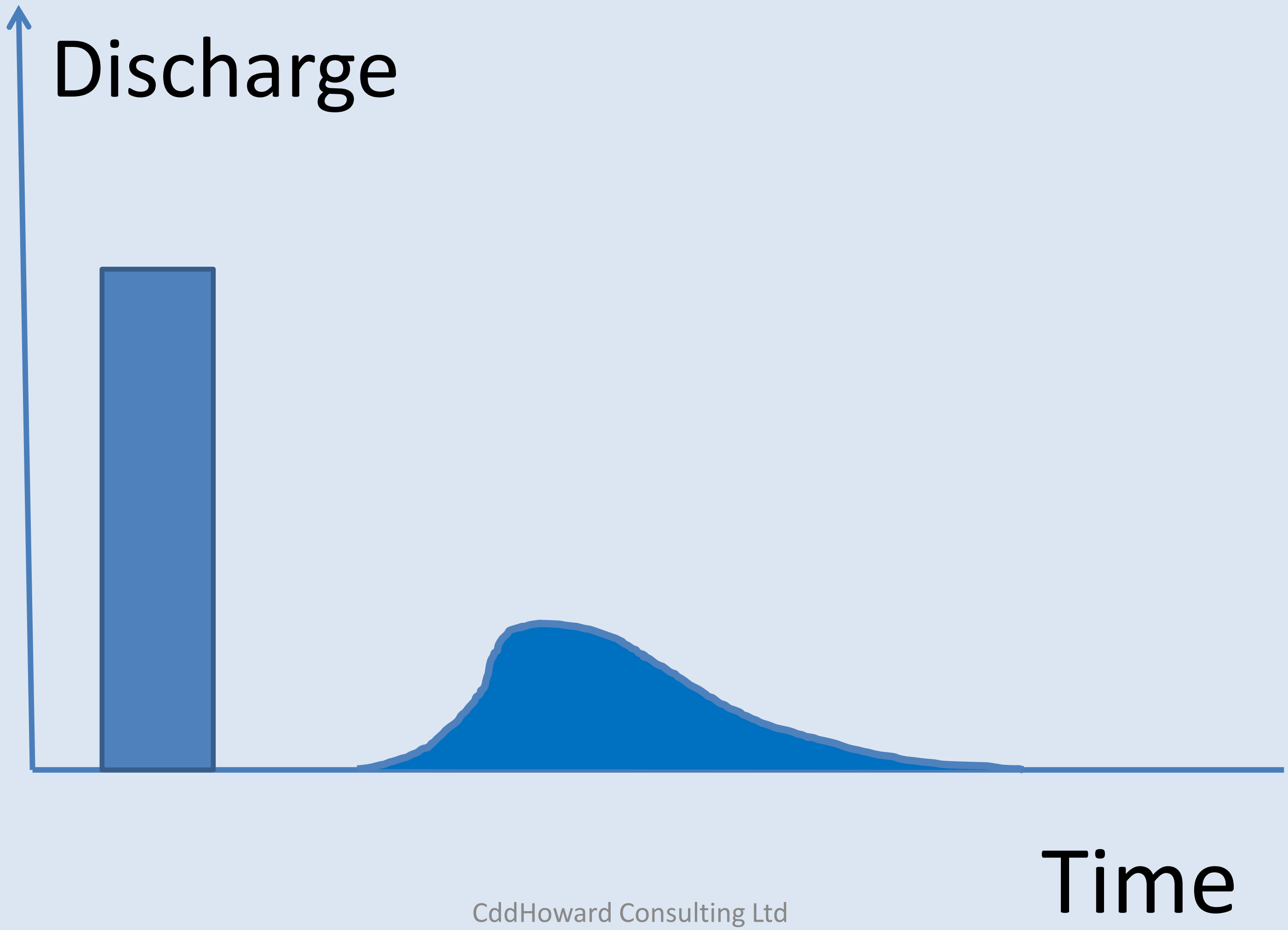
River Routing by Convolution

What Happens in a River



Why this Happens

- Water runs downhill, the more water the steeper the hill must be to keep it moving.
- A sudden release from the dam immediately increases the water surface slope at the dam.
- Some of the water is stored in the river channel as the level rises.
- The rest of the water moves on downstream, and again an increment is stored in the channel.
- Throughout this process the water surface slope is getting flatter, so the rate of discharge is decreasing exponentially.
- The exponential decrease in discharge is what we see in the hydrograph.

How This Happens

- Initial flow in the channel and local inflows
- Initial slope of the river
- Width of the channel
- Hydraulic roughness of the channel
- Depth of the river (lake)
- Obstructions in the channel
- Volume and rate of the release from the dam

Methods of River Routing

- Study methods based on hydraulic engineering equations and channel dimensions and roughness.
- Field observations and judgement
- Theoretical equations based on linear systems analysis - Convolution.

Convolution

$$Q(x,t) = \int_{o}^{t} q\left(0, t-\tau\right) \quad h\left(x,\tau\right) d\,\tau$$

Discharge Q, at downstream location x , at time t is determined by summing the product of the dam release q at x=0 and the routing function h at location x. The term t – τ allows early dam releases to contribute to the hydrograph at all later times.

The summation takes place over times from 0 to the "t" of interest. The integration variable, τ, accounts for the lag and attenuation of the hydrograph caused by channel storage.

The routing function is a unit function that has a total mass of unity – thus the total volume of the release from the dam is not changed by the routing.

How it computes

Q(1) = Discharge downstream at time 1
h(1) = Unit impulse response at τ = 1, q(0,t- τ = 0)

At time 1 $Q(1) = q(1)h(0) + q(0)h(1)$

At time 2 $Q(2) = q(2)h(0) + q(1)h(1) + q(0)h(2)$

At time 3 $Q(3) = q(3)h(0) + q(2)h(1) + q(1)h(2) + q(0)h(3)$

and so on

$Q(n) = q(n)h(0) + q(n-1)h(1) \ldots. + q(1)h(n-1) + q(0)h(n)$

River routing in an LP Optimization

- Notice that the convolution equations can be used as equality constraints in a linear program (LP).

- Therefore in an LP optimization the routing effects of decisions can be incorporated directly into the formulation of the dam's operating constraints.

- The optimization can decide on timed releases that exactly match downstream requirements, like maximum and minimum flows and ramp rates at downstream control points.

Example

$Q(1) = q(1)h(0) + q(0)$ = Downstream flow at time 1

$Q(2) = q(2)h(0) + q(1)h(1) + q(0)h(2)$ = ditto at time 2

$Q(3) = q(3)h(0) + q(2)h(1) + q(1)\, h(2) + q(0)\, h(3)$ = etc.

Flows downstream reflect releases from upstream at all previous times.

How do we find $h(x, \tau)$

- By observation. Observe a sudden release from upstream and the changes in water level downstream.
- Use the observations in a calibration spreadsheet and adjust a trial $h(x, \tau)$ function until good agreement is reached.
- Since convolution is a linear process, and river routing is non-linear, there are sets of $h(x, \tau)$ to cover the flow range of interest.

UIR for a long uniform rectangular canal

The following equation was derived theoretically[1]

$$h_q(x,t) = \frac{1}{2\,(\pi\,D)^{1/2}} \frac{x}{t^{3/2}} \exp\left[-\frac{(ct-x)^2}{4\,Dt}\right]$$

Both c = the advective velocity (Celerity), and the D = Diffusivity, can be estimated by calibration. They both require calibration and depend on the initial flow in the channel and probably on the volume and rate of the release from the dam.

From kinematic wave relationship c = 1.5 (initial discharge/representative depth).

Diffusivity is inversely proportional to river bed slope angle:
D = 0.5 (initial discharge)/slope

[1]Harley, B. M.: Linear Routing in Uniform Open Channels," M. Eng. Thesis, Dept Civil Eng. University of Cork, Ireland, 1967

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