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Technical Basis for Optimizing Hydropower Operations with MS-Excel

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Presented by

John C. Howard, Sr. Systems Analyst
CddHoward Consulting Ltd, Victoria, BC, Canada
<http://www.cddhoward.com>

Abstract

This paper describes techniques which were used to construct an optimization model for hydropower operations using MS-Excel with a third party quadratic programming add-in. The approach takes advantage of MS-Excel's well known convenience for flexibility in data management, model prototyping, and graphical and tabular interfaces. The method determined plant and unit operations to maximize revenue, and meet load and environmental requirements. Aspects of applying linear system's analysis to hydropower operations and approaches to mitigating non-linear aspects will also be addressed. The method was used to model a single dam with six generating units but can be extended to model a larger system. It is expected that the use of convenient and flexible tools such as this may expedite more widespread routine use of optimization methods for improving hydro operating schedules.

Introduction

This paper describes the formulation of a hybrid two stage hourly hydropower operations optimization model that is built around Microsoft Excel [1] and the Frontline Systems' Premium Solver [2]. The former was chosen for its convenient interface and easy formula management abilities and the latter for its ability to solve many kinds of mathematical programming problems, including the quadratic linear programming type that is used to optimize hydropower operations [3, 4].

The goal is to provide a convenient capability to quickly analyze hourly operational scenarios over a time horizon of three years (approximately 25,000 hours) with significant consideration to run time. The approach taken is to build a linear model of a hydropower plant and reservoir, optimize hourly generation value produced by this model with linear programming over one a one week window (168 hours) and correct the hourly output with a non linear post processor to address the linear approximations made in the linear optimization. If only small adjustments are necessary and they can be calculated without adding noticeably to the computer time, the combination of the two methodologies can quickly recommend objective and near optimal operating regimes that avoid the subjective considerations in determining rule curves.

The formulation in Excel passes output from the first stage of the model to the second stage directly via cell values in the spreadsheet for all hours of each weekly run. This is a practical method because the reservoir storage usually is fully utilized within one week. Runs for a multiple week time horizon have input and output managed through script code written in Excel's VBA script language. In a multi week optimization seasonal operating constraints, stored as dated rules in a table, are applied and changed by the script. As the time horizon advances this script also moves the initial conditions forward.

Physical System

The hydroelectric project under study consists of a reservoir with thirteen meters of live storage, a six unit power plant with an installed capacity of over one GW, a tailrace that varies over a range of ten meters, and a reservoir operating range of over forty meters. Inflow is highly regulated with lowest values in the late summer and fall. The average annual project inflow from one year to the next varies by about a factor of two and the annual peak discharge varies by about a factor of eight. There are dams upstream and downstream, but

their hydro plants are not considered in the model. There are no significant tributary inflows between any of the three dams in this cascade.

With the plant generating at full capacity the reservoir has live storage for approximately ten hours. Even with above average inflows this storage can be depleted in a day or so. For this reason, and to incorporate the weekly load cycle, a time horizon of seven-days with an end of week refill target is considered in the model.

The power plant was originally constructed with four generating units with later expansion to include an additional two high capacity units. Management of dissolved gasses in the tailwater require operating these larger units at a lower scheduling priority.

The model considers minimum flows and reservoir and tailbay limits on drawdown rates (down-ramp). Tailbay level is also affected by transient waves in the downstream river channel.

The project creates revenue from energy sales on the market while ensuring some amount of capacity in reserve. The project operates to maximize generation value; off-peak energy and on-peak reserve capacity can be purchased at costs that are less than on-peak energy prices. This design reduces the linear programming model complexity and execution time.

Model Formulation

The model has two-stages: the first stage is a quadratic optimization model which determines the hourly total powerhouse discharge that maximizes generation revenue within the operating constraints. The constraints and functions in the optimization model are linear but the objective function is quadratic; making a quadratic linear program, QLP. The second stage is a non-linear postprocessor, NLPP, which disaggregates the optimized plant discharge and simulates the operation of the project using the proper non-linear curves that describe the forebay, tailbay and unit efficiencies. The NLPP also includes the generation unit dispatch and scheduling optimization algorithm, tailored to the specific operational requirements of this project, to ensure that the total plant discharge is utilized optimally at each hour.

In the equations below, QLP decision variables are **boldface** and input variables appear are *italicized*. Subscripts t designate the 168 time steps; subscripts u designate the 6 power plant units. In some cases variables make use of both subscripts.

Quadratic Linear Program

The QLP model in Excel is formulated with the linear functions and intermediate calculations written in columns and time steps located in rows of a spreadsheet. Each cell formula for the model references a linear combination of constant values, other linear calculations, or cells specially marked as being decision variables for the optimization. When given a reference to the objective function cell the Frontline Solver analyzes the formula relationships and attempts to maximize the objective function. When solved, the values of the decision variables are written back to the spreadsheet. This allows subsequent calculations made from those cells, and the second stage part of the model, to continue.

Constraint Set

The basic formulation of the QLP ensures the correct mass balance within each time step and from one to the next. This considers the stage-storage curve of the reservoir, the tailbay stage-discharge curve, the initial reservoir elevation, the effect of head on plant discharge capacity and spill, and any derived initial conditions such as tailbay level, computed from initial plant discharge and spill.

To begin, we write equations (1) and (2) for water balance through the reservoir and power plant, including spill. This relates change in storage to inflow and total outflow. From these equations we can write the equations for forebay (3) and tailbay (4) elevation using best fit linear slopes calculated from the nonlinear curves.

$$(1) \Delta\text{Storage}_t = \text{In}Q_t - \text{Out}Q_t$$

$$(2) 0 = \text{Power}Q_t + \text{Spill}Q_t - \text{Out}Q_t$$

$$(3) \text{FBElevationDatum} = \text{FBElevation}_t - \text{Storage}_t * \text{FBSlope}$$

$$(4) \text{TBElevationDatum} = \text{TBElevation}_t - \text{Out}Q_t * \text{TBSlope}$$

The relations that calculate head (5) and generation (6) are also straightforward linear equations. In computing generation a dimensionally correct coefficient is used to convert energy in combination with an optimistic estimated aggregate plant efficiency factor of ninety two percent.

$$(5) 0 = \text{Head}_t - \text{FBElevation}_t + \text{TBElevation}_t$$

$$(6) 0 = \text{Generation}_t - 0.92 * C0 * \text{Head}_t * \text{Power}Q_t$$

Generation value (7) is calculated using factors for unit conversion and dimensionless scaling. This provides an hourly price pattern for the week. Maximizing the sum of this generation value over all 168 hours is the primary term in the objective function.

$$(7) 0 = \text{Value}_t - C1 * \text{Generation}_t * \text{SellPrice}_t$$

Within the limits of the hard constraints there are preferred target values set by environmental or other criteria. These preferred operational targets, typically called soft constraints, are formulated in the QLP as a 'difference from target value'. Mathematically, this difference is expressed using two positive terms which are, themselves decision variables: a term expressing a deficit, or shortfall, from the target and a term expressing a surplus from the target value. Formulating the soft constraints in this way permits the inclusion of the constraint into the objective function and allows constraint prioritization with the addition of a weighting factor.

The soft-end-of week storage target is formulated (8) in a way that relates the model's end storage and the target storage using the "two term difference method" described above. Both terms are included in the objective function to make the ending target storage a soft constraint and penalize the objective function when the ending storage deviates from this target value.

$$(8) \text{StorageTarget}_{168} = \text{Storage}_{168} + \text{StorageDeficit}_{168} - \text{StorageSurplus}_{168}$$

The equations that facilitate soft ramping constraints on the forebay (9) and tailbay (10) are written similarly. In this formulation the two difference terms are expressed as a change in storage that effectively causes a change in reservoir elevation. With this formulation elevation down-ramps are represented with the deficit terms. When constrained with target values these terms are used to apply a penalty in the objective function for down-ramp events.

$$(9) (\text{FBDownrampTargetElevation}_t / \text{FBSlope}) = \Delta\text{Storage}_t + \text{FBStorageDeficit}_t - \text{FBStorageSurplus}_t$$

$$(10) (\text{TBDowrampTargetElevation}_t / \text{TBSlope}) = \Delta\text{Out}Q_t + \text{TBQDeficit}_t - \text{TBQSurplus}_t$$

The equation that formulates the soft constraint for minimum discharge (11) is also written using the two term difference method. In this formulation the deficit expresses shortfalls in the target minimum discharge.

$$(11) \text{Min}Q_t = \text{Out}Q_t + \text{MinQDeficit}_t - \text{MinQSurplus}_t$$

The model allows no exceptions to hard constraints, such as bounds on operating ranges, so they must be chosen carefully by considering the physical system and realistic capabilities of the control facilities. Typical constraints include absolute limits on the reservoir minimum and maximum storage as well as maximum outflows and generation from the power station.

Hourly generation targets and reserves were not directly considered constraints inside the QLP model. The amounts to which these commitments are met directly by plant generation are calculated after the QLP optimization. They are a deficit in generation or reserve and are accommodated through optional purchases. Purchases are costs when calculating the final generation value from the QLP. This formulation has the effect of keeping the constraint set smaller and linear as well as eliminating potential mathematical infeasibilities.

Objective Function

The QLP objective function (12) determines the sequence of plant discharges that maximize total generation revenue within hard constraints. Solutions are shaped by energy value as well as guided towards desirable ramp rates and target levels by penalty terms, representing the model's soft constraints, in the objective function. These terms have carefully chosen coefficients that prioritize the constraints. These kinds of constraints result in hourly generation patterns typical of those shown in figure 1.

$$(12) \text{ Maximize } \{ (\Sigma \text{ Value}_t) - C2*(\Sigma \text{ SpillQ}_t) - C3*(\Sigma \text{ MinQDeficit}_t) - C4*(\Sigma \text{ FBStorageDeficit}_t) - C5*(\Sigma \text{ TBQDeficit}_t) - C6*(\text{StorageDeficit}_{168}) - C7*(\text{StorageSurplus}_{168}) \}$$

Non-Linear Post Processor

The QLP model requires linear approximations to tailbay, stage-storage, and efficiency curves. The linear approximations are not acceptable when the QLP output suggests operating on the high curvature ranges of the stage-storage, tailwater or unit efficiency curves. The non-linear post processor (NLPP) uses the optimized plant discharge output to dispatch and load the generating units and to correct the forebay and tailbay elevation values determined by the QLP.

The NLPP on the Excel spreadsheet is laid out with a structure similar to that of the QLP model; columns are used for different types of calculations and rows are used for the sequence of time steps. The QLP solution arrives into the cells for the decision variables and NLPP then relies upon Excel to execute the formulas which depend upon those values.

The forebay stage-storage curve is represented by a quadratic function (13). Figure 2 illustrates a typical week of plant operation showing that the model operates through the week to meet the end-of-week target level of 606.5 m.

$$(13) \text{ FBElevation}_t = A1 + A2 * \text{Storage}_t + A3 * \text{Storage}_t^2$$

Transient wave effects in the tailbay due to sudden changes in plant discharge can have an effect on total head and reduce the accuracy of the NLPP computed value for generation. Tailbay elevations were improved by calibrating a theory based [5] unit impulse response function (14) to create a convolution of the total plant discharge (15).

$$(14) h_t = (\text{distance} / [2 * (\pi * \text{diffusivity})^{0.5} * t^{1.5}]) * (e^{-(\text{velocity} * t - \text{distance}) * [(\text{velocity} * t - \text{distance}) / 4] * t * \text{diffusivity}})$$

$$(15) \text{ SmoothQ}_t = \Sigma (\text{OutQ}_{t - \tau} * h_{\tau}) * d\text{Tau}$$

The tailwater curve is also represented by a quadratic function (16) that is a function of the smoothed project discharge. Figure 2 shows a week's operation with the tailbay ramp-rate constrained to a -5cm limit.

$$(16) \text{ TBElevation}_t = B1 + B2 * \text{SmoothQ}_t + B3 * \text{SmoothQ}_t^2$$

A site specific approach in the NLPP determines generating unit dispatch and loading for each of the six units in the power station. The total station discharge determined by the QLP is allocated to generating units in priority order up to the maximum capacity of each before bringing on additional units and performing efficiency adjustments. The units of lowest priority are also subject to a minimum discharge requirement if the model decides to use them.

Three of the six units have similar efficiency curves and the remaining three have different curves. Each efficiency curve is represented as a 6th order polynomial (17) for each unit and is used to directly calculate megawatts from discharge for each timestep. Actual power generation is corrected for the current head.

$$(17) MW_{tu} = (K1_u + K2_u * PowerQ_{tu} + K3_u * PowerQ_{tu}^2 + K4_u * PowerQ_{tu}^3 + K5_u * PowerQ_{tu}^4 + K6_u * PowerQ_{tu}^5 + K7_u * PowerQ_{tu}^6) * (Head_t / DesignHead_u)^{1.5}$$

Conclusions

An Excel spreadsheet provided several advantages during development of the QLP model. It was very easy to move within the dataset to inspect values and formulas and moderately convenient to use the Excel tools that permit sheet debugging. Debugging could have been easier if there was a way to toggle the sheet view to 'view formulas' instead of cell values.

The VBA scripting language in Excel was very useful for formulating the non linear parts of the NLPP. VBA provided the ability to extend the basic spreadsheet function library to include custom functions that would have been unwieldy to write in a cell or impossible in the case of the unit scheduling or data management routines. VBA development was mostly painless except for quirks with the Excel Object Model.

The Premium Solver plug-in for the QLP model worked well for our needs. The optimization only contained 1346 variables and the plug-in supported up to 5000. On a 2.4 GHZ computer a typical 168 hour run with the plug-in required approximately 15 seconds of setup time and 2 seconds to generate a solution. Solution time appeared to depend upon the hydrology and constraints chosen for the run; longer runtimes were experienced for periods of average hydrology as well as for constraint configurations that were more flexible. These results would indicate that it may be practical to use a similar model formulation for a larger system, consisting of more than one hydro project, or for a larger time horizon, perhaps optimizing hourly over a month.

The linear stage storage approximation fitted to the true stage-storage curve tended to underestimate water in storage at high and low elevations, while slightly over estimating in the middle ranges. The underestimation of storage at the high elevations resulted in the QLP being conservative in its operation and avoiding spill when the linear approximation indicated high reservoir levels. NLPP made the appropriate correction and correctly calculated the ramp rates that only could be approximated in the QLP.

Soft constraints eliminated problems of numerical infeasibility and provided an ability to prioritize and fine tune operations by adjusting the penalties. However, the formulation used in this model is unable to distinguish between high and low density constraint violations so violations can only be detected in aggregate over the 168 time steps. Further research may reveal a practical method to distinguish penalty distributions within the time steps of the model.

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References

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Figures

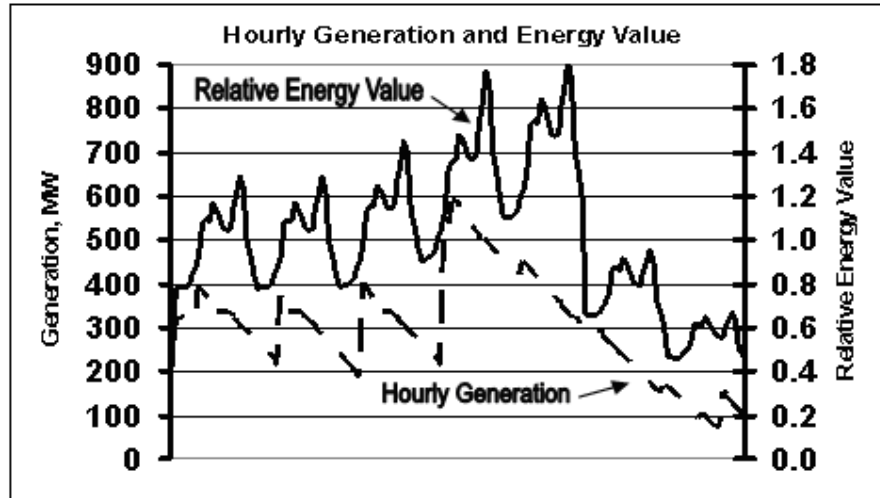


Figure 1: Hourly generation determined by the model generally follows the shape of hourly energy value but is tempered by operational constraints.

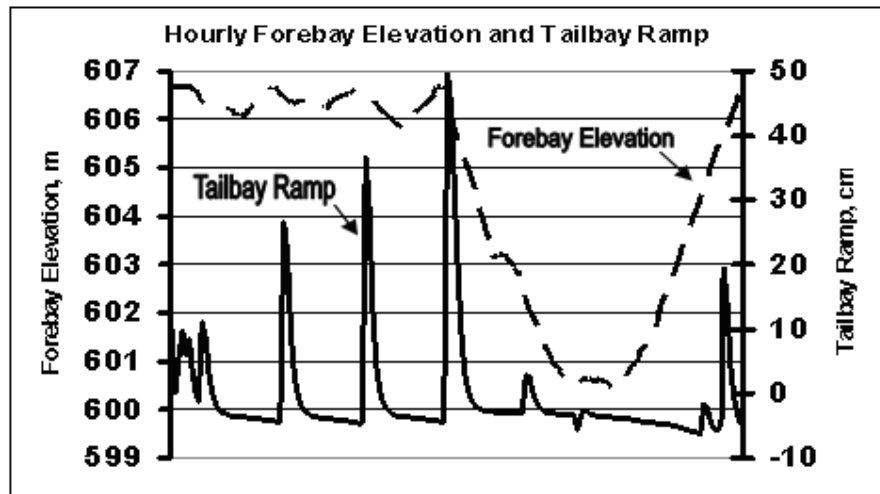


Figure 2: The model optimizes generation value with respect to the forebay and tailbay ramp-rates, the forebay end-of-week target level and other operational constraints.